

Aircraft fleet optimization by the simplex method

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Abstract. The transport system efficiency and the quality of its operation is organically connected with the improvement of air transportation technical means on the basis of modern science and technology. One of the directions of further aviation transport system development is the creation of methodological basis for optimal aircraft operation. Therefore, the increase in aircraft efficiency is one of the important tasks, as its improvement is associated with savings in material and financial resources, which affects the fleet structure and the process of passenger traffic. The article is intended to substantiate the method of optimizing the use of airline aircraft by means of mathematical apparatus. It has been found that the mathematical model structure is required to form an optimal aircraft fleet. A variant of aircraft fleet formation has been proposed taking into account the mathematical model, built on the basis of linear programming, namely, using the simplex method. It has been determined that the choice of optimal aircraft options is greatly influenced by the value of the parametric series. To obtain a reliable result, the aircraft loading must be 100%, otherwise the error grows due to coefficients at unknown target function. The proposed method of determining the optimal fleet with different types of aircraft and air traffic routes, taking into account the conditions of their usage, allows to recommend airlines to use aircraft with the highest efficiency.

1 Introduction

Civil aviation plays an important role in the development of the country's economy, the solution of socio-economic problems, and is of special importance for Ukraine as a means of providing long-range international passenger traffic. The growing needs of the market and the lack of management strategies to satisfy them contribute to the emergence of a new conceptual approach to determining the relationships between its participants. Based on the conditions of the civil aviation market operation, the research of its development processes

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ceases to be fragmentary and acquires a comprehensive study focus in developing the scientific basis for integration of its structural elements [1].

When studying the civil aviation market, it is necessary to use a system approach, which allows to consider the object as a system and focuses the research on disclosing its integrity, to identify various links in it and bring them together into a single complex [5-17]. A system approach permits to comprehensively describe the essence of the civil aviation market and give effective definitions of basic concepts.

Methodological and structural study of a complex air transport system requires large-scale research in all major fundamental areas. Moreover, the role of the initial stage of the structure synthesis is increasing, since at this stage most of the conceptual and structural decisions must be made, that will determine the efficiency and cost-effectiveness of the future aircraft fleet structure [2]. The breadth of the tasks is explained with the great variety of aircraft types and combination of air traffic routes.

Substantiation of the qualitative and quantitative composition of the aircraft fleet, its basing by airline groups, ensuring the aircraft rational use during operation is associated with a large number of factors, the main of which are the following:

- aircraft performance and operational-economic characteristics;
- extent of air routes;
- location of airports along the estimated airlines network;
- volume and time characteristics of passenger and cargo flows;
- restrictions by the flights frequency on airlines;
- airports technical and technological capabilities for aircraft receiving and maintenance;
- airport production complexes capacity;
- aircraft flight safety requirements;
- environmental protection requirements.

The above factors can be combined into two general groups: characteristics determining the aircraft type and the aircraft operating conditions [11].

The aircraft performance and economic data such as: weight, take-off and landing speed characteristics; passenger capacity and maximum commercial load-carrying capacity; flight range (maximum and close to maximum pay-load capacity); technological effectiveness of technical and commercial services; perfect system of pilotage and navigation equipment; specific fuel consumption and other indicators of the aircraft type determine the possibility of its use in the specified operating conditions [10; 8]. When developing new types of aircraft, their performance and economic characteristics are substantiated taking into account the system of the second group factors and their possible change in the calculation period. The second group factors directly affect the distribution of aircraft types in operation by airlines and base airports. When substantiating new aircraft types, the factors of both the first and the second groups may change. According to the current aircraft fleet, only the factors determining the operating conditions are changeable [14].

The extent of the route is one of the main factors determining the choice of aircraft for operation on the airline. Comparison of distances by airlines and route sections with the economical aircraft flight range permits to ground rational areas of their use by the range in order to approximate the airline fixed for each aircraft type to its economical range and multiple of its size (at intermediate landings). The airline extent affects the choice of aircraft in the following way:

- if the distance of the route without intermediate landings is close to the economical range of the aircraft type, its operation is effective and appropriate;
- if the route distance significantly exceeds the economical range, but intermediate landings are possible in areas close to the economical range, and the total extent of the

airline differs slightly from the value multiple of the optimal range, the aircraft operation is effective and possible;

- if the distance along the airline and route sections exceeds significantly the economical range, the operation of the aircraft is inexpedient;
- if the distance along the routes is less than the economical range, the possibility of aircraft operation is assessed by specific deviations and the level of demand for transportation.

The volume and time characteristics of passenger traffic and passenger – cargo flows are associated with the volume of shipments and its distribution over the periods of the year, which determines the possible flight frequency of a particular aircraft type. If the aircraft is suitable for a separate airline, but does not provide the recommended minimum flight frequency at the allowable loading level, its use on the airline is inexpedient. Each of the above factors separately affects the choice of aircraft for operation on the airline, but the final decision can be developed only by taking into account the extent of the flight route, passenger – cargo traffic, allowable values of aircraft loading level and flight frequency.

When substantiating the typical aircraft composition for airlines and planning the use of the aircraft fleet, all groups of factors are considered together. Flight safety and economic factors are a priority. To ensure the necessary operation conditions for the calculated aircraft types, design measures are being developed to promote the airline network efficiency and appropriate airport equipment.

In our country, civil aviation has always played a particular role. Based on the study of a number of scientific papers, we have attempted to implement an approach to determining the required airline aircraft fleet from the standpoint of a single indicator of the air transport system components effectiveness [6; 9].

The efficiency of the air transport system and the quality of its operation is organically connected with the improvement of air transportation technical means on the basis of modern science and technology. One of the directions of further air transport system development is the creation of methodological basis for optimal aircraft operation [23]. Therefore, increasing the aircraft efficiency is one of the important tasks, as its improvement is associated with savings in material and financial resources affecting the fleet structure and the process of passenger traffic [3].

Optimization is the choice of the best option from the many possible ones. If the selection criterion is known and there are few options, the solution can be found by searching and comparing all the options. In our case, the number of possible options is so great that a complete search is almost impossible. So, we have to formulate problems in the language of mathematics and apply special methods to find the optimal solution, i.e. optimization methods [15].

When conducting a research, scientists focus on the optimal organization of passenger and freight flows to maximize the demand for transportation at minimal costs. Such studies are conducted by Savchenko L.V. (2002), Dudukalov Y.V., Zabara S.S., Dekhtyaruk M.T., however, the impact of the combination of aircraft types on the cost of transportation is not taken into account.

In their scientific works, researchers Shmelyova T.F., Sikirda Y.V., Gaevsky S.V. (2021) present a transport model allowing to find the optimal aircraft distribution along selected routes with a minimum air transportation cost under the conditions of compliance with the restrictions on passenger traffic volumes, the number and passenger capacity of aircraft available. The purpose of the article is to substantiate the method of airline fleet optimization using a mathematical apparatus.

2 Methodology

All optimization tasks are divided into two classes:

- 1) mathematical programming problems or static problems;
- 2) optimal control tasks or dynamic tasks [20].

The difference between these classes is that in the problem of mathematical programming it is necessary to find the optimal number, while in the problem of optimal control - the optimal function. From a formal-mathematical point of view, this difference is significant, but in our applied case, it is conditional, and we can ignore it.

In our case, the task is to determine the most efficient aircraft (AC), but it should be borne in mind that the choice of optimal aircraft options is greatly influenced by the value of parametric series, therefore, we have to determine their optimal number, ensuring passenger traffic volumes in due time, as well as the most rational distribution of traffic routes for each aircraft type [21].

The solution of the above complex problem should answer the following questions:

- 1) what types of aircraft must be used for a given volume of passenger traffic?
- 2) on which routes is it advisable to use each type of aircraft?
- 3) how many aircraft do the airlines need to perform passenger traffic?

The above task assumes that passenger transportation is carried out with full 100% loading and the established regularity of flights. Let's suppose that it is necessary to carry out air transportation "n" routes from a certain point "A", as shown in figure 1 below.

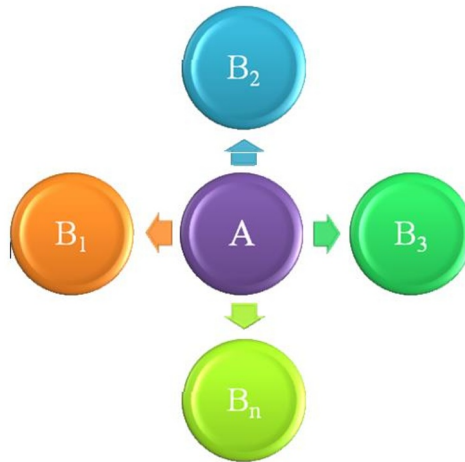


Fig. 1. Air transportation scheme

Let's assume that these routes are served by the following types of aircraft:

$$1, 2, \dots, m \tag{1}$$

where m is the type of aircraft.

Suppose the number of air passengers who need to be carried along each route for a certain period of time - a week, a month, etc. is known.

Let us denote the number of air passengers carried as b_1 - the number of air passengers carried on the 1-st route; b_2 - the number of air passengers carried on the 2-nd route; b_n - the number of air passengers carried on the n -th route [20; 7].

The number of flights performed on the 1-st route by aircraft of the first type is denoted as $X_{1,1}$.

The number of flights operated on the 2-nd route by aircraft of the first type will be indicated $X_{1,2}$.

Accordingly, the number of flights performed on the i -th route by aircraft of the j -th type, is denoted as $X_{i,j}$, where $i = 1, 2, \dots, n; j = 1, 2, \dots, m$.

The number of passengers carried on one flight on the i -th route by aircraft of the j -th type, is denoted as $a_{i,j}$, where $i = 1, 2, \dots, n; j = 1, 2, \dots, m$.

The costs of one flight on the i -th route of the j -th type aircraft is denoted as $c_{i,j}$, where $i = 1, 2, \dots, n; j = 1, 2, \dots, m$.

Suppose each route is served by aircraft of all types $1, 2, \dots, m$. Then for the first route the number of passengers carried b_1 is calculated by the formula:

$$b_1 = a_{1,1} \cdot X_{1,1} + a_{1,2} \cdot X_{1,2} + \dots + a_{1,m} \cdot X_{1,m} \quad (2)$$

For the second route, the number of passengers carried b_2 is calculated by the formula:

$$b_2 = a_{2,1} \cdot X_{2,1} + a_{2,2} \cdot X_{2,2} + \dots + a_{2,m} \cdot X_{2,m} \quad (3)$$

Finally, for all routes a system of constraints-equalities is formed:

$$\begin{cases} b_1 = a_{1,1} \cdot X_{1,1} + a_{1,2} \cdot X_{1,2} + \dots + a_{1,m} \cdot X_{1,m} \\ b_2 = a_{2,1} \cdot X_{2,1} + a_{2,2} \cdot X_{2,2} + \dots + a_{2,m} \cdot X_{2,m} \\ \dots \\ b_n = a_{n,1} \cdot X_{n,1} + a_{n,2} \cdot X_{n,2} + \dots + a_{n,m} \cdot X_{n,m} \end{cases} \quad (4)$$

The total amount of expenses for all flights of all routes z , is calculated by the formula:

$$\sum_{i=1}^n \sum_{j=1}^m c_{ij} \cdot X_{ij} \quad (5)$$

If it is necessary to minimize the total amount of costs according to the formula (5) when performing the system of constraints-equalities (4), we get the problem of linear programming. The problem of linear programming is solved by the simplex method, i.e. by this method $X_{i,j}$ is found, where $i = 1, \dots, n; j = 1, \dots, m$.

After determining $X_{i,j}$, it is possible to determine the optimal aircraft fleet for the "A" node. It is possible to add a system of restrictions-equalities (inequalities) to the system of restrictions-equalities according to the number of flights of each aircraft type, as a result of which we get the total number of flights on all routes of aircraft type 1.

To determine the rational use of investment, it is necessary to conduct a feasibility study of the hypothetical aircraft fleet structure by airlines, their geographical location and the

country as a whole. Productivity growth and passenger traffic volume must outpace the cost, which is a prerequisite for the efficient use of each new aircraft [1].

The aircraft is characterized by a large number of parameters - weight, geometric, aerodynamic. When optimizing the aircraft fleet, it is necessary to link the aircraft parameters with the process of passenger traffic, the more efficient the connection, the more cost-effective the process will be.

From the above, it follows that to carry out the process of passenger transportation at minimal costs, it is necessary to have aircraft that can be estimated by developing a parametric series based on the forecast of domestic and world aircraft construction. The input data for calculation of parametric series can be set for a certain, rather long prospective period conditioned by planning output indicators. Therefore, for each subsequent period, new parametric series should be developed related to the conditions of the new period.

The solution of the perspective parametric series optimization problem is connected with consideration of a complex dynamic stochastic system. The complexity of the system is determined by a large number of interdependent factors in the system. The dynamic nature of this system lies in the fact that during a long period determined by the parametric series, there is a change in the passenger traffic volume, the conditions of its implementation and, consequently, the number of aircraft in the fleet. However, there are difficulties in accounting the factors related to aircraft and obtaining necessary information quantitatively characterizing these factors. All this suggests the need at this stage to consider a static task for a certain period of time [4].

To construct the parametric series is to determine the basic aircraft characteristics and the optimal number of aircraft on each route by means of the mathematical method. The selection of the optimal aircraft fleet, taking into account the air traffic tactical and technical characteristics, the number of passengers in the aircraft, as well as the number of flights per month and the i -th route costs per month, is made comprehensively using the simplex-method. The information that can be obtained using the simplex-method is not limited to the optimal values of variables. The simplex-method allows to give an economic interpretation of the obtained solution and to analyze the model for sensitivity. Sensitivity model analysis is a process implemented after the optimal solution is obtained. Such an analysis reveals the optimal solution sensitivity to certain changes in the original model [16].

The process of solving the linear programming problem by the simplex method is iterative, i.e., computational procedures of the same type are repeated in a certain sequence until the optimal solution is obtained. Simplex-method procedures naturally require the use of computers. In the simplex-method computational scheme, an ordered process is implemented, where starting from some initial admissible point, successive transitions are made from one admissible extreme point to another, until a point (type of aircraft on the route) corresponding to the optimal solution is found.

The starting point for the optimal aircraft fleet is the initial decision. From the starting point, there is the transition to the next adjacent point. The choice of the point depends on the target function coefficients: where the coefficient is higher the function is subject to maximization, which leads to an extreme point. This process is then repeated to find out if there is another extreme point corresponding to a better solution. As a result, this iterative process permits to find the optimal aircraft for the corresponding route [19; 22].

The simplex algorithm consists of the following stages:

1) using a standard linear model, determine the initial admissible basic solution by setting non-basic variables to zero;

2) among the current non-basic (zero) variables, a variable is selected that is included into the new basis, the increase of which improves the target function value. If there is no

such variable, the calculation is stopped as the current basic solution is optimal. Otherwise, there is the transition to the next stage;

3) among the variables of the current basis, a variable to be excluded is selected, which must be set to zero (become non-basic) when introducing a new variable into the basic group;

4) there is a new basic solution corresponding to the new non-basic and basic variables composition. The transition to stage "2" is underway.

From the theoretical issues underlying the simplex method construction, it follows that the angular point is fully determined by the basic solution of the linear programming problem. The conditions of the simplex algorithm optimality and admissibility provide the transition from the initial admissible point (initial basic solution) to the adjacent point, which corresponds to the improved target function value.

3 Results

According to Table 1, the total number of passengers carried on route 1 per month is $b_1 = 61488$ people.

Table 1. Initial data for aircraft fleet optimization by linear programming method

Route	Aircraft Type	Number of passengers in aircraft	Number of passengers per month	Number of flights per month	V_{cr} , km/h	L_p , km	t_p , one aircraft, h	Costs on i -th route per month, USD
1	Aircraft 1	120	4480	84	910	1225	4	23621818
	Aircraft 2	76	6720	84	850		4	40727273
	Aircraft 3	170	2800	84	800		4.33	33506327
2	Aircraft 1	120	1280	8	910	1750	6	10123636
	Aircraft 3	170	1600	16	800		6	26530909
3	Aircraft 1	120	2208	8	910	1470	4.67	7879564
4	Aircraft 3	170	1536	8	800	1512	5	11054545
5	Aircraft 1	120	640	4	910	3150	8	6749091
6	Aircraft 1	120	1280	8	910	1785	3	5061818
	Aircraft 2	76	1920	8	850		3	8727273
7	Aircraft 1	120	1280	8	910	1680	3	5061818
	Aircraft 2	76	960	4	850		3	4363636

Enter the letter designation:

- X_1 – the number of aircraft 1 flights on route 1 per month;
- X_2 – the number of aircraft 2 flights on route 1 per month;
- X_3 – the number of aircraft 3 flights on route 1 per month.

The number of passengers carried by one flight on route 1 is:

- Aircraft 1 - 240 passengers;
- Aircraft 2 - 152 passengers;
- Aircraft 3 - 340 passengers.

As a result, the linear constraint-equality for route 1 is presented in the following form:
 $240X_1 + 152X_2 + 340X_3 = 61488$.

Similarly, make calculation for all seven routes. Then, determine the relative cost of aircraft 2 and aircraft 3 flight relative to aircraft 1.

For aircraft 2 the cost is 0.63, for aircraft 3 - 1.43.

Taking into account the above number of flights per month for the considered aircraft types on all routes according to Table 1 and taking into account the prices of one flight for

each aircraft type on the considered routes, the target function of total costs for all flights on all routes of the hypothetical airline is proposed in the following form:

$$z = 236.2X_1 + 335X_2 + 407.3X_3 + 101.2X_4 + 265.3X_5 + 145X_6 + 78.8X_7 + 50X_8 + 113X_9 + 175.5X_{10} + 110.5X_{11} + 251X_{12} + 67.5X_{13} + 42.5X_{14} + 96.5X_{15} + 50.6X_{16} + 32X_{17} + 87.3X_{18} + 50.6X_{19} + 32X_{20} + 43.6X_{21}$$

For this target function, the system of constraints-equalities according to the equations is presented in the following form:

$$\begin{cases} 240X_1 + 152X_2 + 340X_3 = 61488 \\ 240X_4 + 152X_5 + 340X_6 = 11712 \\ 240X_7 + 152X_8 + 340X_9 = 5856 \\ 240X_{10} + 152X_{11} + 340X_{12} = 5856 \\ 240X_{13} + 152X_{14} + 340X_{15} = 2928 \\ 240X_{16} + 152X_{17} + 340X_{18} = 11712 \\ 240X_{19} + 152X_{20} + 340X_{21} = 8784 \end{cases} \tag{6}$$

Solutions by means of the basic simplex-method

The output table is as follows (table 2).

Table 2. The output simplex table

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	C	
																							236.2
																							335
																							407.3
																							101.2
																							265.3
																							145
																							78.8
																							50
																							113
																							175.5
																							110.5
																							251
																							67.5
																							42.5
																							96.5
																							50.6
																							32
																							87.3
																							50.6
																							32
																							43.6
																							0
																							61488
																							11712
																							5856
																							5856
																							2928
																							11712
																							8784

Calculate the deltas:

$$\Delta_i = C_1 \cdot a_{1i} + C_4 \cdot a_{2i} + C_7 \cdot a_{3i} + C_{10} \cdot a_{4i} + C_{13} \cdot a_{5i} + C_{16} \cdot a_{6i} + C_{19} \cdot a_{7i} - C_i$$

Table 3. Simplex table with deltas

C	236.2	335	407.3	101.2	265.3	145	78.8	50	113	175.5	110.5	251	67.5	42.5	96.5	50.6	32	87.3	50.6	32	43.6	0	
basic	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	X21	b	
X1	1	0.63333	1.41666	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	256.2
X4	0	0	0	1	0.63333	1.41666	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	48.8
X7	0	0	0	0	0	0	1	0.63333	1.41666	0	0	0	0	0	0	0	0	0	0	0	0	0	24.4
X10	0	0	0	0	0	0	0	0	0	1	0.63333	1.41666	0	0	0	0	0	0	0	0	0	0	24.4
X13	0	0	0	0	0	0	0	0	0	0	0	0	1	0.63333	1.41666	0	0	0	0	0	0	0	12.2
X16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.63333	1.41666	0	0	0	0	48.8
X19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.63333	1.41666	0	36.6
Δ	0	-	-	0	-	-	0	-	-	0	0.65	-2.375	0	0.25	0.875	0	0.04666	-	-	0	0.04666	28.08333	76802.66

Determine the column-column resolution with the maximum delta:

21, Δ21: 28.08333

Let's find the simplex-relations Q by dividing the coefficients b by the corresponding values of column 21.

In the found column, look for the line with the smallest Q value:

$Q_{min} = 25.83529$, line 7.

At the intersection of the found line and column there is the resolution: 1.41666

Let's take X_{21} as the basic variable X_{19} .

Table 4. Iteration 1

C	236.2	335	407.3	101.2	265.3	145	78.8	50	113	175.5	110.5	251	67.5	42.5	96.5	50.6	32	87.3	50.6	32	43.6	0	Q	
basic	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	X21	b		
X1	1	0.63333	1.41666	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	256.2	-
X4	0	0	0	1	0.63333	1.41666	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	48.8	-
X7	0	0	0	0	0	0	1	0.63333	1.41666	0	0	0	0	0	0	0	0	0	0	0	0	0	24.4	-
X10	0	0	0	0	0	0	0	0	0	1	0.63333	1.41666	0	0	0	0	0	0	0	0	0	0	24.4	-
X13	0	0	0	0	0	0	0	0	0	0	0	0	1	0.63333	1.41666	0	0	0	0	0	0	0	12.2	-
X16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.63333	1.41666	0	0	0	0	48.8	-
X21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.63333	1.41666	36.6	36.6/1.41666=25.83529	
Δ	0	-	-	0	-	-	0	-	-	0	0.65	-2.375	0	0.25	0.875	0	0.04666	-	-	0	0.04666	28.08333	76802.66	

Divide line 7 by 1.41666. From lines 1, 2, 3, 4, 5, 6 subtract line 7 multiplied by the corresponding element in column 21.

Calculate new deltas:

$$\Delta_i = C_1 \cdot a_{1i} + C_4 \cdot a_{2i} + C_7 \cdot a_{3i} + C_{10} \cdot a_{4i} + C_{13} \cdot a_{5i} + C_{16} \cdot a_{6i} + C_{21} \cdot a_{7i} - C_i$$

$$\Delta_1 = C_1 \cdot a_{11} + C_4 \cdot a_{21} + C_7 \cdot a_{31} + C_{10} \cdot a_{41} + C_{13} \cdot a_{51} + C_{16} \cdot a_{61} + C_{21} \cdot a_{71} - C_1 = 236.2 \cdot 1 + 101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 236.2 = 0$$

$$\Delta_2 = C_1 \cdot a_{12} + C_4 \cdot a_{22} + C_7 \cdot a_{32} + C_{10} \cdot a_{42} + C_{13} \cdot a_{52} + C_{16} \cdot a_{62} + C_{21} \cdot a_{72} - C_2 = 236.2 \cdot 0.63333 + 101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 335 = -185.40666$$

$$\Delta_3 = C_1 \cdot a_{13} + C_4 \cdot a_{23} + C_7 \cdot a_{33} + C_{10} \cdot a_{43} + C_{13} \cdot a_{53} + C_{16} \cdot a_{63} + C_{21} \cdot a_{73} - C_3 = 236.2 \cdot 1.41666 + 101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 407.3 = -72.68333$$

$$\Delta_4 = C_1 \cdot a_{14} + C_4 \cdot a_{24} + C_7 \cdot a_{34} + C_{10} \cdot a_{44} + C_{13} \cdot a_{54} + C_{16} \cdot a_{64} + C_{21} \cdot a_{74} - C_4 = 236.2 \cdot 0 + 101.2 \cdot 1 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 101.2 = 0$$

$$\Delta_5 = C_1 \cdot a_{15} + C_4 \cdot a_{25} + C_7 \cdot a_{35} + C_{10} \cdot a_{45} + C_{13} \cdot a_{55} + C_{16} \cdot a_{65} + C_{21} \cdot a_{75} - C_5 = 236.2 \cdot 0 + 101.2 \cdot 0.63333 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 265.3 = -201.20666$$

$$\Delta_6 = C_1 \cdot a_{16} + C_4 \cdot a_{26} + C_7 \cdot a_{36} + C_{10} \cdot a_{46} + C_{13} \cdot a_{56} + C_{16} \cdot a_{66} + C_{21} \cdot a_{76} - C_6 = 236.2 \cdot 0 + 101.2 \cdot 1.41666 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 145 = -1.63333$$

$$\Delta_7 = C_1 \cdot a_{17} + C_4 \cdot a_{27} + C_7 \cdot a_{37} + C_{10} \cdot a_{47} + C_{13} \cdot a_{57} + C_{16} \cdot a_{67} + C_{21} \cdot a_{77} - C_7 = 236.2 \cdot 0 + 101.2 \cdot 0 + 78.8 \cdot 1 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 78.8 = 0$$

$$\Delta_8 = C_1 \cdot a_{18} + C_4 \cdot a_{28} + C_7 \cdot a_{38} + C_{10} \cdot a_{48} + C_{13} \cdot a_{58} + C_{16} \cdot a_{68} + C_{21} \cdot a_{78} - C_8 = 236.2 \cdot 0 + 101.2 \cdot 0 + 78.8 \cdot 0.63333 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 50 = -0.09333$$

$$\begin{aligned} \Delta_9 &= C_1 \cdot a_{19} + C_4 \cdot a_{29} + C_7 \cdot a_{39} + C_{10} \cdot a_{49} + C_{13} \cdot a_{59} + C_{16} \cdot a_{69} + C_{21} \cdot a_{79} - C_9 = 236.2 \cdot 0 + 101.2 \cdot 0 \\ &+ 78.8 \cdot 1.41666 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 113 = -1.36666 \\ \Delta_{10} &= C_1 \cdot a_{110} + C_4 \cdot a_{210} + C_7 \cdot a_{310} + C_{10} \cdot a_{410} + C_{13} \cdot a_{510} + C_{16} \cdot a_{610} + C_{21} \cdot a_{710} - C_{10} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 1 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 175.5 = 0 \\ \Delta_{11} &= C_1 \cdot a_{111} + C_4 \cdot a_{211} + C_7 \cdot a_{311} + C_{10} \cdot a_{411} + C_{13} \cdot a_{511} + C_{16} \cdot a_{611} + C_{21} \cdot a_{711} - C_{11} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0.63333 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 110.5 = 0.65 \\ \Delta_{12} &= C_1 \cdot a_{112} + C_4 \cdot a_{212} + C_7 \cdot a_{312} + C_{10} \cdot a_{412} + C_{13} \cdot a_{512} + C_{16} \cdot a_{612} + C_{21} \cdot a_{712} - C_{12} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 1.41666 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0 - 251 = -2.375 \\ \Delta_{13} &= C_1 \cdot a_{113} + C_4 \cdot a_{213} + C_7 \cdot a_{313} + C_{10} \cdot a_{413} + C_{13} \cdot a_{513} + C_{16} \cdot a_{613} + C_{21} \cdot a_{713} - C_{13} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 1 + 50.6 \cdot 0 + 43.6 \cdot 0 - 67.5 = 0 \\ \Delta_{14} &= C_1 \cdot a_{114} + C_4 \cdot a_{214} + C_7 \cdot a_{314} + C_{10} \cdot a_{414} + C_{13} \cdot a_{514} + C_{16} \cdot a_{614} + C_{21} \cdot a_{714} - C_{14} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0.63333 + 50.6 \cdot 0 + 43.6 \cdot 0 - 42.5 = 0.25 \\ \Delta_{15} &= C_1 \cdot a_{115} + C_4 \cdot a_{215} + C_7 \cdot a_{315} + C_{10} \cdot a_{415} + C_{13} \cdot a_{515} + C_{16} \cdot a_{615} + C_{21} \cdot a_{715} - C_{15} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 1.41666 + 50.6 \cdot 0 + 43.6 \cdot 0 - 96.5 = -0.875 \\ \Delta_{16} &= C_1 \cdot a_{116} + C_4 \cdot a_{216} + C_7 \cdot a_{316} + C_{10} \cdot a_{416} + C_{13} \cdot a_{516} + C_{16} \cdot a_{616} + C_{21} \cdot a_{716} - C_{16} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 1 + 43.6 \cdot 0 - 50.6 = 0 \\ \Delta_{17} &= C_1 \cdot a_{117} + C_4 \cdot a_{217} + C_7 \cdot a_{317} + C_{10} \cdot a_{417} + C_{13} \cdot a_{517} + C_{16} \cdot a_{617} + C_{21} \cdot a_{717} - C_{17} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0.63333 + 43.6 \cdot 0 - 32 = 0.04666 \\ \Delta_{18} &= C_1 \cdot a_{118} + C_4 \cdot a_{218} + C_7 \cdot a_{318} + C_{10} \cdot a_{418} + C_{13} \cdot a_{518} + C_{16} \cdot a_{618} + C_{21} \cdot a_{718} - C_{18} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 1.41666 + 43.6 \cdot 0 - 87.3 = -15.61666 \\ \Delta_{19} &= C_1 \cdot a_{119} + C_4 \cdot a_{219} + C_7 \cdot a_{319} + C_{10} \cdot a_{419} + C_{13} \cdot a_{519} + C_{16} \cdot a_{619} + C_{21} \cdot a_{719} - C_{19} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0.70588 - 50.6 = -19.82352 \\ \Delta_{20} &= C_1 \cdot a_{120} + C_4 \cdot a_{220} + C_7 \cdot a_{320} + C_{10} \cdot a_{420} + C_{13} \cdot a_{520} + C_{16} \cdot a_{620} + C_{21} \cdot a_{720} - C_{20} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 0.44705 - 32 = -12.50823 \\ \Delta_{21} &= C_1 \cdot a_{121} + C_4 \cdot a_{221} + C_7 \cdot a_{321} + C_{10} \cdot a_{421} + C_{13} \cdot a_{521} + C_{16} \cdot a_{621} + C_{21} \cdot a_{721} - C_{21} = 236.2 \cdot 0 + \\ &101.2 \cdot 0 + 78.8 \cdot 0 + 175.5 \cdot 0 + 67.5 \cdot 0 + 50.6 \cdot 0 + 43.6 \cdot 1 - 43.6 = 0 \\ \Delta_b &= C_1 \cdot b_1 + C_4 \cdot b_2 + C_7 \cdot b_3 + C_{10} \cdot b_4 + C_{13} \cdot b_5 + C_{16} \cdot b_6 + C_{21} \cdot b_7 - C_{22} = 236.2 \cdot 256.2 + \\ &101.2 \cdot 48.8 + 78.8 \cdot 24.4 + 175.5 \cdot 24.4 + 67.5 \cdot 12.2 + 50.6 \cdot 48.8 + 43.6 \cdot 25.83529 - 0 = \\ &76077.11882 \end{aligned}$$

At the fourth iteration we obtain the optimal plan according to which the target function is equal to 76043.66513 USD.

The same result is obtained by the electronic system means of aircraft fleet comparative analysis:

$$\{x_1': 256.2, x_2': 0, 'min': 76043.6651393189, x_3': 0, x_4': 48.8, x_5': 0, x_6': 0, x_7': 24.4, x_8': 0, x_9': 0, x_{10}': 0, x_{11}': 38.526315789473685, x_{12}': 0, x_{13}': 0, x_{14}': 19.263157894736842, x_{15}': 0, x_{16}': 0, x_{17}': 77.05263157894737, x_{18}': 0, x_{19}': 0, x_{20}': 0, x_{21}': 25.83529411764706\}$$

4 Conclusion

Based on the calculation results we can draw a conclusion:

- 1) on three routes, namely: 1, 2 and 3, economically efficient is the use of aircraft type 1;
- 2) on three routes, namely: 4, 5 and 6, economically efficient is the use of aircraft type 3;
- 3) on route 7 it is economically efficient to use type 2 aircraft.

To solve optimization problems means to find and achieve the optimal correlation of the air transport system parameters and a number of system parameters affecting the air transport system optimality and performance. Optimality indicates the presence of acceptable properties in the system according to the adopted efficiency criterion. In our case, it is the aircraft fleet structure and dimension in the interaction with the process of

passenger transportation. The following essential characteristics or conditions of optimization problems must be defined:

- optimization criteria in the form of effect indicators, combined into a general criterion of economic efficiency;
- algorithms and methods for estimating a single criterion indicator and checking optimality;
- determination of the general criterion indicator of the passenger transportation process economic efficiency at changing the parameters influencing the change of target function.

As it was mentioned above, airlines are in dire need of modern aircraft. Outdated fleet and changing requirements for aircraft performance are the ground for the use of system analysis and operation research methods. A mathematical model structure is required to form an optimal aircraft fleet. This paper proposes a variant of the aircraft fleet formation taking into account the mathematical model built on the basis of linear programming, by means of the simplex method.

The article substantiates and formulates a method for determining the required aircraft type for a particular route, based on the developed algorithm for selecting the best option by means of the simplex method. The proposed criterion assessment of the required aircraft fleet structure allows to analyze a large number of variants of characteristic parameter combinations using the proven research methods. The developed method of determining the required aircraft fleet structure on the given flight routes takes into account the limited amount of initial data on the relationship and interaction of the selected air transport system characteristics, which involves the use of generalized statistical data or predicted solutions. The total error, in this case, leads to a loss of the accuracy of distribution limit values by routes.

Thus, the above method allows to solve such problems as finding the optimal number of aircraft types in the estimated Airline, determining the distribution of the aircraft number by types, which provides solutions to tasks assigned to a particular air transport system or airline. In order to minimize the total costs, the aviation transport system will be considered as consisting of aircraft subsystems and the process of passenger transportation, which does not contradict the general methodological basis for aircraft fleet optimization.

A feature of the passenger transportation production cycle is the aircraft flight range, which allows the use of short-medium-haul aircraft. When considering the issue of choosing the optimal aircraft fleet, it is necessary to take into account a system approach to goal planning, take account of both the prospects of goal development and its complex solution using most interrelationships within the air transport system and outside it.

The choice of optimal aircraft options is greatly influenced by the parametric series values. To obtain a reliable result, the aircraft loading should be 100%, otherwise the error increases due to unknown target function coefficients. The proposed method of determining the optimal fleet of different aircraft types and air traffic routes, taking into account the conditions of their operation, permits to recommend airlines to use aircraft with the highest efficiency.

It has been found out that it is necessary to input the current generalized schedule of aircraft flights on the routes with the obligatory indication of the aircraft type, its capacity, price and flight time. Next, passenger traffic and the relative cost of flights for other aircraft were calculated, after which a system of restrictions was developed to further minimize the costs. The key factor is the desired passenger traffic. The system will independently select the most efficient aircraft for a particular direction. The proposed software package, in its turn, can solve the problem of comparing the aircraft efficiency in the Airline for the shortest time and it is free to operate.

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Competing interests

The authors declare that they have no competing interests.

Availability of data and materials

Data will be made available on reasonable request.

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