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Construction of a mathematical model of nonlinear deformation of a composite polymer material of a heterogeneous structure

A mathematical model has been constructed to predict the deformation of a composite material, the matrix of which is a polymer material containing dispersed inclusions or pores. The influence of the concentration of inclusions, their shapes and sizes on the deformation process of this material has been taken into account.

The interest in composite materials is caused to the fact that their inherent set of properties and features significantly distinguishes them from homogeneous materials. This is primarily high strength and low density, as well as the ability to control mechanical and physical characteristics when creating a structural material. In particular, composite materials are one of the most innovative and sought-after materials in the aviation industry. They are extremely important in the production of aircraft, providing high strength and lightness of structures, which consequently reduces fuel consumption and reduces the load on the aircraft engine. Composites can be used to create various parts of aircraft, including the fuselage, wings, tail, nose and doors. These materials have high strength and corrosion resistance, which makes them ideal for use in extreme temperatures and humidity. Moreover, composites can be easily molded into any shape, which allows you to create structures with unusual geometry.

The properties of composites depend primarily on the properties of the original components: reinforcing elements and matrix. In addition, their combination gives the effect of synergy, connected with the appearance of properties that are not characteristic of isolated original components. Composites are distinguished by a wide range of useful and unique properties, and their rational combination allows you to obtain effective structures with a high degree of weight perfection and a given anisotropy of the physical and mechanical characteristics of the material.

The mechanical characteristics of the composite are determined by the ratio of the properties of the reinforcing elements and the matrix, as well as the strength of the bonds between them. The effectiveness and productiveness of this material depend on the correct choice of the initial components and the technologies for their combination. These factors must ensure a strong connection between the specified components while preserving their initial characteristics. As a result of the combination of high-strength inclusions and the matrix, a complex of composite properties is formed, which reflects not only the initial characteristics, but also additionally includes properties that the individual components do not have.

Composite binders, the polymers, are known to exhibit creep deformations under the action of stresses , which, concequently, are accompanied by the appearance of microcracks and microvoids at the boundaries of crystalline grains. As a result, the effective cross-sectional area that accepts the load decreases, and the creep rate increases.

Let's consider the deformation of an isolated inclusion of canonical form in an infinite environment with linear or nonlinear viscous properties. Assume, there is an axisymmetric load on a sufficiently distant boundary. The stress concentration on the interfacial surface of the inclusion depending on its shape and the physical and mechanical properties of the matrix and the composite material as a whole should be investigated. The matrix of the composite material is considered to be isotropic and has viscoelastic properties. The dependence of the strain rate $\dot{\varepsilon}_{ij}$ on the stresses σ_{ij} is assumed to be power law, in particular, for one-dimensional tension [1]

$$\dot{\varepsilon} = \dot{\varepsilon}_0(\sigma / \sigma_0)^n \qquad (1)$$

Here $\dot{\varepsilon}_0$ and σ_0 are values of the strain rate and stresses of the initial state. The stiffness index n varies from unity to infinity.

For a three-dimensional stress-strain state, equation (1) takes the form [2]

$$\dot{\varepsilon}_{ij} = \frac{3}{2}\dot{\varepsilon}_0(\sigma_e / \sigma_0)^{n-1} s_{ij} / \sigma_0 ,$$

where s_{ij} is a deviator of the stress tensor, σ_e are effective stresses

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{mm}\sigma_{ij} ; \qquad \sigma_e = \left(\frac{3}{2} s_{ij} s_{ij}\right)^{\frac{1}{2}} .$$

Hence, $\dot{\varepsilon}_{ij} = \frac{1}{2\eta}\sigma_e^{n-1} s_{ij}$, where η is viscosity parameter, which is determined by the relationship [3]

$$\eta = \frac{\sigma_0^n}{3\dot{\varepsilon}_0} .$$

These relations define both materials of linear viscous Newtonian solid at n=1 and rigid-plastic Prandtl solid at $n \to \infty$. Let us consider inclusions of axisymmetric shape and such that they can be described by the equation

$$\frac{x_1}{b^2} + \frac{x_2}{b^2} + \frac{x_3}{a^2} = 1 .$$

The stress at a sufficiently distant boundary ∂B of the region B of the composite material is denoted by: $\sigma_{11} = \sigma_{22} = T$; $\sigma_{33} = S$; $x \in \partial B$, where the coordinate system coincides with the principal axes of the spheroid.

Thus, if we denote $\sigma = S - T$,

then $\qquad \sigma_e(x) = |\sigma| , \qquad x \in \partial B$

$$\dot{\varepsilon}_{33}(x) = \dot{\varepsilon} ; \qquad \dot{\varepsilon} = \frac{1}{3\eta}|\sigma|^{n-1}\sigma , \qquad x \in \partial B .$$

Consider a linear problem

$$\sigma_{ij} = \lambda_{ijkl}\, \dot{\varepsilon}_{kl} . \qquad (2)$$

Let us write law (2) in the form [4]

$$\sigma_{ij} = 2\eta(e_{ij} + \frac{\nu}{1-2\nu} e_{mm}\sigma_{ij}) ,$$

where ν is Poisson's ratio.

If the tensor Q relating the stress σ_{ij}^{∞} at the remote boundary and the strain rate of the inclusion [5] is introduced

$$\bar{\sigma}_{ij} = Q_{ijkl} e_{kl}^{i} ; \qquad \bar{\sigma}_{ij} = \sigma_{ij}^{\infty} ,$$

then

$$Q_{ijkl} = \lambda_{ijkl} - \lambda_{ijmn} S_{mnkl} .$$

Here S_{ijkl} is an Eschebi tensor. If we denote *w=a/b* the ratio of two semi-axes [8], then

$$\beta = I_b / (2\pi) = \begin{cases} w(1-w^2)^{-3/2}[\arccos w - w(1-w^2)^{1/2}], & w<1 \\ w(w^2-1)^{-3/2}[w(w^2-1)^{1/2} - arcch w], & w>1 \end{cases} .$$

To determine the local stress distribution at the boundary of a spheroidal inclusion in a multicomponent material, a solution constructed by the theory of potentials and integral transformations was used [4,5].

The obtained equations are true for arbitrary types of spatial reinforcement. The derivation of calculation formulas is connected with the specification of an explicit form of the density of the distribution of inclusions in directions. Expressions for the tensor coefficients of stress concentrations in inclusions and the matrix material of the composite material are:

$$\sigma_i = K_{ci}(H)\sigma^M , \qquad \sigma_m = K_{cm}(H)\sigma^M ,$$

where

$$K_{ci}(H) = \lambda_i \frac{A_i \lambda^{-1} + B_i(\tilde{E}_{(1)}, H_{(1)})}{\sigma^M} ;$$

$$K_{cm}(H) = \lambda_m \frac{A_m \lambda^{-1} + B_m(\tilde{E}_{(1)}, H_{(1)})}{\sigma^M} . \qquad (3)$$

Here σ^M is stress at the remote boundary of a given element. Operator relations (3) contain rational functions of integral viscoelasticity operators.

Conclusion.

Thus, a mathematical model of the deformation of composite materials was constructed taking into account the dependence of stresses on inclusions that are randomly placed in the medium and have a spheroidal shape. Spherical and disk-shaped inclusions were considered as particular cases. The long-term strength of composite products depends on the magnitude of the average or maximum stresses per cycle of loading in the matrix material and in the inclusions, the number of cycles, etc. In this regard, the work studies microstructural stresses, calculates effective parameters: and determines their dependence on the shape, orientation and volumetric concentration of inclusions.

References

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